Space–time symmetries in duality symmetric models.[*]

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Talk given at the workshop on "Gauge theories, applied supersymmetry and quantum gravity"
(Leuven, Belgium, 10–14 July, 1995)

Abstract

A way of covariantizing duality symmetric actions is described. As examples considered are a manifestly space–time invariant duality–symmetric action for abelian gauge fields coupled to axion–dilaton fields and gravity in D=4, and a Lorentz–invariant action for chiral bosons in D=2. The latter is shown to admit a manifestly supersymmetric generalization for describing chiral superfields in n=(1,0) D=2 superspace.

We would like to report on a covariantization of duality symmetric actions in various space–time dimensions considered earlier by

Zwanziger (1971) for Maxwell fields in D=4 [1];

Floreanini and Jackiw (1987) for chiral bosons in D=2 [2];

Henneaux and Teitelboim (1987) for self–dual tensor fields in $D = 4p + 2$ (p=1,2,...)[3];

Tseytlin (1990) for a duality symmetric string [4];

Schwarz and Sen (1993) for dual fields in any D [5, 6].

In connection with recent progress in understanding the important role of duality symmetries in Yang–Mills theories [7, 8, 9], supergravity and string theory [10]–[14], the construction of versions of these theories where duality would be a manifest symmetry of the action may help to gain new insight into the structure of these theories.

A particular feature of duality symmetric models considered so far [2]–[6] is that the presence of duality symmetry violates manifest Lorentz and general coordinate invariance and supersymmetry of the action, conventional space–time symmetries being replaced by some modified transformations of fields.

However, usually it is desirable to have a space–time covariant formulation, which makes the structure of the theory more transparent and often reveals new properties and new links between different parts of the theory.

So, one can admit that these duality symmetric actions are, in fact, a gauge choice in more general manifestly space–time and duality symmetric models with richer local symmetry structure [15, 16, 17]. We shall show that this is indeed the case by use of D=4 Maxwell theory and a D=2 chiral field model as examples.

1   Duality in Maxwell theory

It is well known that the action for a free Maxwell field $A_m(x)$ (m=0,1,2,3)

$$S = - \int d^4x \frac{1}{4} F_{mn} F^{mn} = \frac{1}{2} \int d^4x (\mathbf{E}^2 - \mathbf{B}^2) \tag{1}$$

is not invariant under duality transformations of the electric and magnetic strength vectors $\mathbf{E} \rightarrow \mathbf{B}$, $\mathbf{B} \rightarrow -\mathbf{E}$, while the free Maxwell equations are invariant:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}, \qquad \nabla \mathbf{E} = \nabla \mathbf{B} = 0. \tag{2}$$

To have duality symmetry at the level of action one has to double the number of gauge fields (A_m^α, $\alpha = 1, 2$) [1, 18, 5] and construct an action in such a way that one of the gauge fields becomes an auxiliary field upon solving equations of motion [5]. An

alternative duality symmetric version of Maxwell action was considered in [19]. The duality symmetric action of refs. [1, 5] can be written in the following form:

$$S = \int d^4x \left(-\frac{1}{8} F^\alpha_{mn} F^{mn\alpha} + \frac{1}{4} \mathcal{F}^\alpha_{0i} \mathcal{F}^\alpha_{i0}\right), \qquad (i = 1, 2, 3) \tag{3}$$

where

$$\mathcal{F}^\alpha_{mn} = \mathcal{L}^{\alpha\beta} F^\beta_{mn} - \frac{1}{2} \epsilon_{mnlp} F^{lp\alpha} = \frac{1}{2} \epsilon_{mnlp} \mathcal{F}^{lp\beta} \mathcal{L}^{\alpha\beta}, \tag{4}$$

$(\mathcal{L}^{12} = -\mathcal{L}^{21} = 1)$.

Duality symmetry is a discrete subgroup of $SO(2)$ rotations of A^α_m ($A^\alpha_m \rightarrow \mathcal{L}^{\alpha\beta} A^\beta_m$).

Note that $\mathcal{F}^\alpha_{mn} \mathcal{F}^{\alpha mn} = 0$ due to the self–duality property, and the second term in (3) breaks manifest Lorentz invariance. However, beside the manifest spacial rotations the action (3) is invariant under the following modified space–time transformations of A^α_i (in the gauge $A^\alpha_0 = 0$)

$$\delta A^\alpha_i = x^0 v^k \partial_k A^\alpha_i + v^k x^k \partial_0 A^\alpha_i + v^k x^k \mathcal{L}^{\alpha\beta} \mathcal{F}^\beta_{0i}, \tag{5}$$

where the first two terms describe the ordinary Lorentz boosts along a constant velocity v^i and the third term vanishes on the mass shell since an additional local symmetry of the action (3)

$$\delta A^a_0 = \varphi^\alpha(x) \tag{6}$$

allows one to reduce the equations of motion

$$\frac{\delta S}{\delta A^\alpha_i} = \epsilon^{ijk} \partial_i \mathcal{F}^\alpha_{k0} = 0 \tag{7}$$

to a duality condition

$$\mathcal{F}^\alpha_{mn} = \mathcal{L}^{\alpha\beta} F^\beta_{mn} - \frac{1}{2} \epsilon_{mnlp} F^{lp\alpha} = 0 \tag{8}$$

which, on the one hand, leads to the Maxwell equations

$$\partial_m \mathcal{F}^{mn\alpha} = \partial_m F^{mn\alpha} = 0 \tag{9}$$

and, on the other hand, completely determines one of the gauge fields through another one. For instance, using the relation

$$\frac{1}{2} \epsilon_{mnlp} F^{2mn} = F^1_{mn}$$

we can exclude $A^2_m(x)$ from the action (3) and get the conventional Maxwell action.

Covariantization. One can admit that the action (3) arose as a result of some gauge fixing which specifies time direction in a Lorentz covariant action [16, 17]. [1]

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[1] An attempt to construct this action was undertaken by Khoudeir and Pantoja [15]. Unfortunately, they did not manage to find a consistent formulation of the model.

The first step is to covariantize the self–dual part of the action (3). The simplest possible way to do this is to introduce an auxiliary vector field $u_m(x)$ and write the action as follows:

$$S_A = \int d^4x(-\frac{1}{8}F^\alpha_{mn}F^{\alpha mn} + \frac{1}{4(-u_l u^l)}u^m \mathcal{F}^\alpha_{mn}\mathcal{F}^{\alpha np}u_p).\tag{10}$$

The main problem is to find a local symmetry which would permit to choose a gauge where u_m is a constant vector, in particular,

$$u_m(x) = \delta^0_m.\tag{11}$$

Then the action (10) can reduce to (3). Note that we have introduced the norm of u_m ($u^l u_l = u^2$) into the action (10). This is necessary for ensuring the symmetry we are looking for. When u_m is a constant vector, it corresponds to the frozen straight Dirac string [1], while when $u_m(x)$ depends on space–time coordinates, one can regard the Dirac string to be curved.

The search for this symmetry turns out to be connected with another problem, namely, the problem of preserving a local symmetry under (6). In the covariant version this transformation should be replaced by

$$\delta A^\alpha_m = u_m \varphi^\alpha.\tag{12}$$

To keep this symmetry is important (as we have already seen) for getting the duality condition (8).

To have the invariance under transformations (12) one should add to the Lorentz invariant action (10) another term

$$S_B = -\int d^4x \epsilon^{mnpq}u_m \partial_n B_{pq},\tag{13}$$

where $B_{mn}(x)$ is an antisymmetric tensor field. Then the variation of (10) under (12) is canceled by the variation of (13) under

$$\delta B_{mn} = -\frac{\varphi^\alpha}{u^2}(\mathcal{F}^\alpha_{mp}u^p u_n - \mathcal{F}^\alpha_{np}u^p u_m).\tag{14}$$

Note that (13) is also invariant under

$$\delta B_{mn} = \partial_{[m}b_{n]}(x).\tag{15}$$

As in the case of action (3), the local symmetry (12) allows one to fix a gauge on the mass shell in such a way that the duality condition (8) takes place.

Thus, we again remain with only one independent Maxwell field and get the duality between its electric and magnetic strength vector. In view of the vanishing condition for the self–dual strength tensor the equations of motion of u_m reduce to:

$$\frac{\delta(S_A + S_B)}{\delta u_m} = \epsilon^{mnlp}\partial_n B_{lp} = 0 \quad \to \quad B_{mn} = \partial_{[m}b_{n]},\tag{16}$$

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which means that B_{mn} is completely auxiliary and can be eliminated by use of the corresponding local transformations (15).

The only thing which has remained to show is that u_m itself does not carry physical degrees of freedom and can be gauge fixed to $u_m = \delta_m^0$. For this we have to find a corresponding local symmetry. The form of the action (13) prompts that $S_A + S_B$ ((10), (13)) can be invariant under the following transformations of u_m:

$$\delta u_m(x) = \partial_m \varphi(x). \tag{17}$$

This is indeed the case provided A_m^α and B_{mn} transform as follows

$$\delta A_m^\alpha = \frac{\varphi(x)}{u^2} \mathcal{L}^{\alpha\beta} \mathcal{F}_{mn}^\beta u^n, \qquad \delta B_{mn} = \frac{\varphi(x)}{(u^2)^2} \mathcal{F}_m^{\alpha r} u_r \mathcal{F}_n^{\beta s} u_s \mathcal{L}^{\alpha\beta}. \tag{18}$$

Then, taking into account that the equations of motion of B_{mn} give

$$\partial_{[m} u_{n]} = 0 \quad \rightarrow \quad u_m(x) = \partial_m \hat{\varphi}(x) \tag{19}$$

and requiring that $u^2 \neq 0$ (to escape singularities), we can use this local transformation to put $u_m = \delta_m^0$. In this gauge the manifestly Lorentz invariant duality symmetric action

$$S = \int d^4x \left(-\frac{1}{8} F_{mn}^\alpha F^{\alpha mn} + \frac{1}{4(-u_l u^l)} u^m \mathcal{F}_{mn}^\alpha \mathcal{F}^{\alpha np} u_p - \epsilon^{mnpq} u_m \partial_n B_{pq}\right). \tag{20}$$

reduces to (3), and the local transformations of A_m^α (18) (with $\varphi(x) = x^i v^i$) are combined with the corresponding Lorentz transformations and produce the modified space–time symmetry (5) of the action (3).

It seems of interest to understand the origin of the fields $u_m(x)$ and $B_{mn}(x)$ and of the local transformations (17).

The form of the term in the action (20) containing B_{mn} reminds a term one encounters in a formulation of a pseudoscalar ('axion') field (see, for the details [20, 21, 22, 23, 24] and references therein)

$$S = \int d^4x \left(-\frac{1}{2}(\partial_m a(x) - u_m(x))(\partial^m a(x) - u^m(x)) - \epsilon^{pqmn} u_p \partial_q B_{mn}\right). \tag{21}$$

The action (21) is invariant under local Peccei–Quinn transformations

$$\delta a(x) = \varphi(x), \qquad \delta u_m = \partial_m \varphi(x), \tag{22}$$

(u_m being the corresponding gauge field) and produces dual versions of the axion action:

$$L = -\frac{1}{2} \partial_m a(x) \partial^m a(x),$$

$$L = \frac{1}{3!} \partial_{[m} B_{np]} \partial^{[m} B^{np]}. \tag{23}$$

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The duality relation between the pseudoscalar field $a(x)$ and the antisymmetric tensor field B_{mn}

$$\partial_l a(x) = \epsilon_{lmnp} \partial^m B^{np} \tag{24}$$

is a consequence of the equations of motion of u_m obtained from (21).

Thus, one may treat the origin of the duality symmetric Maxwell action as a result of a specific coupling of the two Maxwell fields to the auxiliary gauge field u_m from the axionic sector of the theory.

$SL(2, \mathbf{R})$ **invariant axion–dilaton coupling and coupling to gravity**.
The duality symmetric model considered above admits axion–dilaton coupling which respects global $SL(2, \mathbf{R})$ invariance as well as coupling to gravity [5, 16, 17].

To do this we introduce a 2×2 matrix dilaton–axion field

$$\mathcal{M} = \frac{1}{\lambda_2(x)} \begin{pmatrix} 1 & \lambda_1(x) \\ \lambda_1(x) & \lambda_1^2 + \lambda_2^2 \end{pmatrix}, \qquad \mathcal{M}^T = \mathcal{M}, \qquad \mathcal{M}\mathcal{L}\mathcal{M}^T = \mathcal{L}, \tag{25}$$

and define the global $SL(2, \mathbf{R})$ transformations as follows:

$$\mathcal{M} \;\rightarrow\; \omega^T \mathcal{M} \omega, \qquad \omega \mathcal{L} \omega^T = \mathcal{L}, \qquad A_m = \omega^T A_m. \tag{26}$$

The coupling is carried out by the modification of the self–dual tensor (4) in the following way

$$\mathcal{F}_{mn}^\alpha = \mathcal{L}^{\alpha\beta} F_{mn}^\beta - \frac{1}{2\sqrt{-g}} \epsilon_{mnlp} (\mathcal{L}^T \mathcal{M} \mathcal{L})^{\alpha\beta} F^{lp\beta} \equiv \frac{\sqrt{-g}}{2} (\mathcal{L}\mathcal{M})^{\alpha\beta} \epsilon_{mnpq} \mathcal{F}^{\beta pq}, \tag{27}$$

where $g = \det g_{mn}(x)$ is the determinant of a gravitational field metric.

In the most simple form the $SL(2, \mathbf{R})$ invariant action is written as follows:

$$S = \int d^4 x \sqrt{-g} \left(\frac{1}{2u^2} u^m F_{mn}^{*\alpha,a} \mathcal{F}^{\alpha,a \; np} u_p - \frac{1}{4} g^{mn} tr(\partial_m \mathcal{M} \mathcal{L} \partial_n \mathcal{M} \mathcal{L}) + \frac{1}{\sqrt{-g}} \epsilon^{pqmn} u_p \partial_q B_{mn} + R \right). \tag{28}$$

Upon fixing the gauge $u_m = \frac{1}{\sqrt{-g^{00}}} \delta_m^0$, $B_{mn} = 0$, the action (28) directly reduces to a corresponding Schwarz–Sen action [5].

Note that we would like to identify the field $a(x)$ with $\lambda_1(x)$ from the axion–dilaton matrix (25), then, due to the specific coupling of the scalars to the abelian gauge fields the local Peccei–Quinn transformations of the axion field are broken down to the global shifts which are part of the $SL(2, \mathbf{R})$ group. This is reflected in the structure of the kinetic term for the scalar fields in (28) which does not contain the auxiliary gauge field u_m. However, the action is still invariant under the local transformations of u_m, A_m^α and B_{mn} (17), (18). One can go even further and eliminate from the action (28) the term containing B_{mn} by substituting into (28) $u_m = \partial_m \hat{\varphi}(x)$.

The action (28) can be extended to include $O(6, 22)$ scalar fields corresponding to a low energy bosonic sector of a toroidally compactified heterotic string in a straightforward way [5, 17].

Adding fermions and supersymmetry.

The kinetic term for neutral fermions is added to the duality symmetric action of a free Maxwell field without any problems:

$$S = \int d^4x (-\frac{1}{8} F^{\alpha}_{mn} F^{\alpha mn} + \frac{1}{4(-u_l u^l)} u^m \mathcal{F}^{\alpha}_{mn} \mathcal{F}^{\alpha np} u_p - \epsilon^{mnpq} u_m \partial_n B_{pq} - i\overline{\psi}\gamma^m \partial_m \psi). \quad (29)$$

This action is supersymmetric [5, 16, 17] [2] under the following transformations with odd constant parameters $\epsilon^{\alpha} = i\gamma_5 \mathcal{L}^{\alpha\beta} \epsilon^{\beta}$:

$$\delta A^{\alpha}_m = i\overline{\psi}\gamma_m \epsilon^{\alpha},$$

$$\delta\psi = \frac{1}{8} F^{\alpha mn}\gamma_m\gamma_n\epsilon^{\alpha} - \frac{1}{4u^2}\mathcal{L}^{\alpha\beta} u_p \mathcal{F}^{\alpha pm} u^n \gamma_m\gamma_n\epsilon^{\beta}, \quad (30)$$

all other fields being inert under the supersymmetry transformations.

We see that the supersymmetric transformation law for the fermion $\psi(x)$ (30) is non–conventional and reduces to the ordinary SUSY transformations of the vector supermultiplet $(A^1_m(x), \psi(x))$ only on the mass shell (8) upon excluding A^2_m:

$$\delta A^1_m = i\overline{\psi}\gamma_m\epsilon^1; \qquad \delta\psi = \frac{1}{4} F^{1mn}\gamma_m\gamma_n\epsilon^1. \quad (31)$$

This is analogous to the problem with the Lorentz transformations (5) which we have just solved. Using the same reasoning as lead us to introducing $u_m(x)$ one may try to find a superpartner of $u_m(x)$ whose presence in the theory gives rise to a local fermionic symmetry (being a counterpart of (17, 18)) which involves $\psi(x)$ and leads to (30) upon gauge fixing the local fermionic symmetry.

We shall demonstrate the existence of such local fermionic symmetry in a simpler model for supersymmetric chiral fields in two space–time dimensions.

2 Chiral bosons and fermions in D=2

For comparison let us consider a formulation of the dynamics of the simplest possible self–dual field, namely a chiral boson $\phi(x)$, in two–dimensional space–time. On the mass shell $\phi(x)$ satisfies a self–duality (chirality) condition:

$$\mathcal{F}_m \equiv \partial_m\phi - \epsilon_{mn}\partial^n\phi = 0 = (\partial_0 - \partial_1)\phi = \partial_{--}\phi \quad (32)$$

and describes right–moving modes. $\mathcal{F}_m = \epsilon_{mn}\mathcal{F}^n$, and $(--, ++)$ denote the light–cone vector components.

There are several (classically equivalent) versions of the chiral boson action [25, 2, 26] from which the chirality condition (32) is obtained. A Lorentz covariant action proposed

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[2]A superfield generalization of the non–covariant action (3) was considered earlier in [30].

by Siegel [25] contains the square of the condition $\partial_{--}\phi = 0$ with a corresponding Lagrange multiplier $\lambda_{++++}(x)$:

$$S = \int d^2x \frac{1}{2}(\partial_{++}\phi\partial_{--}\phi - \lambda_{++++}(\partial_{--}\phi)^2). \tag{33}$$

The system described by the action (33) turns out to be anomalous at the quantum level (because of the constraint $(\partial_{--}\phi)^2 = 0$) and requires an additional Wess–Zumino term to cancel the anomaly [27].

By putting $\lambda_{++++} = 1$, which breaks manifest Lorentz invariance, one gets the Floreanini–Jackiw action [2]

$$S = \int d^2x \frac{1}{2}(\partial_{++}\phi\partial_{--}\phi - (\partial_{--}\phi)^2), \tag{34}$$

which describes a quantum mechanically consistent chiral boson system (provided, appropriate boundary conditions on ϕ are imposed) [2, 28], since the constraint $(\partial_{--}\phi)^2 = 0$ does not directly follow from (34). The ordinary Lorentz transformations of $\phi(x)$ are replaced by:

$$\delta\phi = vx_m\epsilon^{mn}\partial_n\phi - vx^1\partial_{--}\phi, \tag{35}$$

where v is a constant parameter, and the first term is the ordinary Lorentz boost. The sum of the actions (34), one for left–movers and one for right–movers, describes a duality symmetric model being a ground for duality symmetric formulation of bosonic string theory [4].

As in the case of Maxwell theory we can restore the manifest Lorentz invariance by introducing into (34) an auxiliary vector field $u_m(x)$. Because of peculiar properties of the two–dimensional model, this can be carried out in two classically equivalent ways. The first possibility is to consider $u_m(x)$ as a unit–norm time–like vector and to write the action in the form

$$S = \int d^2x \frac{1}{2}(\partial_{++}\phi\partial_{--}\phi - u^m\mathcal{F}_m u^n\mathcal{F}_n). \tag{36}$$

Then, because u_m was required to satisfy $u_m u^m = -1$, it contains only one independent component, and one can show that (36) reduces to the Siegel action (33).

Another possibility, which seems to be more appropriate from the quantum point of view, is to construct a Lorentz covariant action by analogy with the action (20):

$$S = \int d^2x \frac{1}{2}\left(\partial_{++}\phi\partial_{--}\phi + \frac{1}{u^2}u^m\mathcal{F}_m u^n\mathcal{F}_n - \epsilon^{mn}u_m\partial_n B\right), \tag{37}$$

where $B(x)$ is an auxiliary scalar field. The action (37) is invariant under the following local transformations:

$$\delta u_m = \partial_m\varphi, \qquad \delta\phi = \frac{\varphi}{u^2}u^m\mathcal{F}_m, \qquad \delta B = \varphi\left(\frac{u^m\mathcal{F}_m}{u^2}\right)^2, \tag{38}$$

which permit to choose the gauge $u_m = \delta_m^0$ on the mass shell, where:

$$\frac{\delta S}{\delta B} = 0 \quad \rightarrow \quad u_m = \partial_m \hat{\varphi}(x), \tag{39}$$

and reduce (37) to (34).

One can simplify (37) by substituting into the action the expression (39) for u_m. Then (37) takes the following form:

$$S = \int d^2x \frac{1}{2} \left(\partial_{++}\phi \partial_{--}\phi - \frac{\partial_{++}\hat{\varphi}}{\partial_{--}\hat{\varphi}}(\partial_{--}\phi)^2 \right), \tag{40}$$

and the transformations (38) reduce to

$$\delta\hat{\varphi} = \varphi(x), \qquad \delta\phi = \frac{\varphi(x)}{\partial_{--}\hat{\varphi}}\partial_{--}\phi. \tag{41}$$

Note that in contrast to the Siegel case (33) the variation of (40) with respect to the auxiliary field $\hat{\varphi}(x)$ does not produce the anomalous constraint $(\partial_{--}\phi)^2 = 0$, since $\hat{\varphi}$ enters (40) under the derivative.

Adding to (37) or (40) the analogous action (containing the same field u_m) for left–movers and taking an appropriate number of the left– and right–moving scalar fields one obtains the duality symmetric formulation of a string [4, 6] with manifest space–time symmetries.

The Lorentz invariant form (40) of the chiral boson action is directly generalized to a supersymmetric case.

Let us consider bosonic superfields $\Phi(x,^{--}, x^{++}, \theta^+) = \phi(x) + i\theta^+\psi_+(x)$ and $\Lambda(x^{--}, x^{++}, \theta^+)$ $= \hat{\varphi}(x) + i\theta^+\chi_+(x)$, which obey the conventional transformation law under global shifts $\delta\theta^+ = \epsilon^+$, $\delta x^{++} = i\theta^+\epsilon^+$ and $\delta x^{--} = 0$ in $n = (1,0)$ flat superspace. Then the superfield generalization of (40) is

$$S = \int d^2x d\theta^+ \frac{1}{2} \left(D_+\Phi\partial_{--}\Phi - \frac{D_+\Lambda}{\partial_{--}\Lambda}(\partial_{--}\Phi)^2 \right) \tag{42}$$

and that of (41) is

$$\delta\Lambda = C(x^-, x^+, \theta^+), \qquad \delta\Phi = \frac{C(x^-, x^+, \theta^+)}{\partial_{--}\Lambda}\partial_{--}\Phi, \tag{43}$$

where $D_+ = \frac{\partial}{\partial\theta^+} + i\theta^+\partial_{++}$, $D_+^2 = i\partial_{++}$ is the supercovariant derivative.

Using the transformations (43) one can gauge fix all components of $\Lambda(x,\theta)$, except $\Lambda|_{\theta=0} = \hat{\varphi}(x)$, to zero and identify the latter with the time coordinate $x^0 = \frac{1}{2}(x^{++} + x^{--})$. Then, upon integrating over θ^+ one gets a component action (considered in [29] as part of a duality invariant string action with non–manifest supersymmetry):

$$S = \int d^2x \left(\frac{1}{2}(\partial_{++}\phi\partial_{--}\phi - (\partial_{--}\phi)^2) - i\psi_+\partial_{--}\psi_+ \right), \tag{44}$$

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which is invariant under the modified Lorentz transformations for the bosonic field $\phi(x)$ (35) and modified supersymmetry transformations:

$$\delta\phi = i\epsilon^+\psi_+; \qquad \delta\psi = i\epsilon^+\partial_{++}\phi - i\epsilon^+\partial_{--}\phi \qquad (45)$$

for the fermionic field ψ_+ analogous to (30).

From (44) one gets that on the mass shell $\phi(x)$ and $\psi_+(x)$ are chiral:

$$\partial_{--}\phi = 0, \qquad \partial_{--}\psi_+ = 0.$$

3   Discussion

The models considered above represent examples of duality symmetric models with manifest space–time symmetries. We have seen that in D=4 duality between two abelian gauge fields arose as a result of their coupling to the auxiliary vector field which can be treated as the gauge field of local Peccei–Quinn symmetry. The structure of the action admits space–time invariant $SL(2, \mathbf{R})$ axion–dilaton coupling, coupling to gravity and supersymmetrization. The example of chiral (self–dual) fields in two space–time dimensions demonstrates the possibility of constructing duality symmetric models in the form which respects both the conventional Lorentz invariance and conventional global supersymmetry.

The generalization of the results obtained to duality symmetric actions for abelian gauge fields in other dimensions [2]–[5] is rather straightforward.

As far as supersymmetry is concerned, it would be of interest to construct a superfield supergravity generalization of the duality symmetric models. The field content of the bosonic sector considered above points to the possible existence of such supergravity though its consistent formulation may turn out to be not an easy problem.

Other challenging problems are the extension of the class of duality symmetric actions with that describing non–abelian gauge fields, and involving into the consideration charged matter. The discussion of problems of coupling electrically and magnetically charged matter fields to duality symmetric electro–magnetic fields the reader may find in [31] and references therein.

References

[1] D. Zwanziger, Phys. Rev. **D3**, 880 (1971).

[2] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987).

[3] M. Henneaux and C. Teitelboim, in Proc. Quantum mechanics of of fundamental systems 2, Santiago, 1987, p. 79; Phys. Lett. **B206**, 650 (1988).

[4] A. Tseytlin, Phys. Lett. **B242**, 163 (1990); Nucl. Phys. **B350**, 395 (1991).

[5] J. H. Schwarz and A. Sen, Nucl. Phys. **B411**, 35 (1994).

[6] J. H. Schwarz and A. Sen, Phys. Lett. **B312**, 105 (1993).

[7] C. Montonen and D. Olive, Phys. Lett. **B72**, 117 (1977); P. Goddard, J. Nuyts and D. Olive, Nucl. Phys. **B 125**, 1 (1977); H. Osborne, Phys. Lett. **B83**, 321 (1979).

[8] M. Halpern, Phys. Lett. **B81**, 245 (1979); Phys. Rev. **D19**, 517 (1979).

[9] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); Nucl. Phys. **B431**, 484 (1994);
N. Seiberg, The power of duality – exact results in 4D SUSY field theory. Preprint RU-95-37, IASSNS-HEP-95/46, hep–th/9506077 (and references therein).

[10] K. Narain, Phys. Lett. **B169**, 41 (1986); K. Narain, H. Sarmadi and E. Witten, Nucl. Phys. **B279**, 369 (1987).

[11] A. Font, L. Ibañez, D. Lust and F. Quevedo, Phys. Lett. **B249**, 35 (1990); S. J. Rey, Phys. Rev. **D43**, 526 (1991).

[12] A. Shapere, S. Trivedi and F. Wilczek, Mod. Phys. Lett. **A6**, 2677 (1991).

[13] A. Sen, Nucl. Phys. **B404**, 109 (1993).

[14] J. H. Schwarz, String theory symmetries. Preprint CALT-68-1984, hep–th/9503127 (and references therein).

[15] A. Khoudeir and N. Pantoja, hep–th/9411235.

[16] P. Pasti, D. Sorokin and M. Tonin, Phys. Lett. **B352**, 59 (1995); hep-th/9503182.

[17] P. Pasti, D. Sorokin and M. Tonin, Phys. Rev. **D52**, R1 (1995), hep-th/9506109.

[18] R. Kallosh and T. Ortin, Phys. Rev. **D48**, 742 (1993).

[19] S. Deser and C. Teitelboim, Phys. Rev. **D13**, 1592 (1976); S. Deser, J. Phys. A: Math. Gen. **15**, 1053 (1982).

[20] V. I. Ogievetsky and I. V. Polubarinov, Soviet. J. Nucl. Phys. **4**, 156 (1967); M. Kalb and P. Ramond, Phys. Rev. **D9** 2273 (1974).

[21] M. J. Bowick, S. B. Giddings, J. A. Harvey, G. T. Horowitz and A. Strominger, Phys. Rev. Lett. **61**, 2823 (1988); S Coleman, J. Preskill and F. Wilczek, Nucl. Phys. **B378**, 175 (1992).

[22] H. Nicolai and P. K. Townsend, Phys. Lett. **B98**, 257 (1981).

[23] R. Kallosh, A. Linde, D. Linde and L. Susskind, Preprint SU-ITP-95-2, hep–th/9502069.

[24] F. Quevedo, Preprint NEIP95–007, hep–th/9506081.

[25] W. Siegel, Nucl. Phys. **B238**, 307 (1984).

[26] R. Amorim and J. Barcelos–Neto, hep–th/9505096.

[27] C. Imbimbo and A. Schwimmer, Phys. Lett. **B193**, 455 (1987).

[28] M. E. V. Costa and H. O. Girotti, Phys. Rev. Lett. **60**, 1771 (1988); F. P. Devecchi and H. O. Girotti, Phys. Rev. **D49**, 4302 (1994).

[29] A. Ali, Preprint IP/BBSR/94–32, hep–th/9406189.

[30] C. Panagiotakopoulos, A. Salam and J. Strathdee, Phys. Lett. **B115**, 29 (1982).

[31] M. Blagojevìc and P. Senjanovìc, Phys. Rep. **157**, 233 (1988).